Generic rigidity of circle maps with breaks.

We prove that $C^r$-smooth ($r > 2$) circle diffeomorphisms with a break, i.e., a single singular point where the derivative has a jump discontinuity, are generically not $C^{1+\varepsilon}$-rigid, for any $\varepsilon > 0$. That is, for almost all irrational $\rho \in (0, 1)$, and every $\varepsilon > 0$, there is a pair of $C^r$-smooth circle diffeomorphisms with a break, with the same rotation number $\rho$ and the same size of the break which are not $C^{1+\varepsilon}$-smoothly conjugate to each other. This result complements our recent proof (joint with K. Khanin) that such maps are generically $C^1$-rigid. It stands in remarkable contrast to the result of J.-C. Yoccoz that $C^r$-smooth circle diffeomorphisms are generically $C^{r-1-\kappa}$-rigid, for any $\kappa > 0$. (Received January 25, 2014)