Morse Theory is based on the idea that a smooth function on a manifold yields data about its topology. Specifically, Morse’s Isotopy Lemma tells us that two sublevels are diffeomorphic provided there are no critical points between the corresponding levels. Further, the index of the Hessian constrains the change in homotopy type caused by a critical point.

Since Riemannian distance functions are not smooth everywhere, critical points and the Hessian cannot be defined in the usual way. In 1977 Grove and Shiohama created a definition of critical point for distance functions and used it to generalize Morse’s Isotopy Lemma to this case. Their generalization had a profound impact on Riemannian geometry. However, without a definition of index, the remainder of Morse Theory cannot be generalized.

I will present a new notion, called sub-index, and show how to use it to gain information about the change in homotopy type caused by critical points of distance functions. (Received January 24, 2014)