We discuss higher degree immersed finite element (IFE) spaces to be used with finite element methods to solve two dimensional second order elliptic interface problems without requiring the mesh to be aligned with the material interfaces. The interpolation errors in the proposed piecewise $p^{th}$ degree spaces yield optimal $O(h^{p+1})$ and $O(h^p)$ convergence rates in the $L^2$ and broken $H^1$ norms, respectively, under mesh refinement. Moreover, a bilinear IFE space is constructed for solving the two dimensional acoustic problem. A partially penalized method is developed for elliptic interface problems and a discontinuous Galerkin method is constructed for solving first order hyperbolic systems. The finite element errors for both methods converge optimally with the proposed higher degree IFE spaces. Several numerical examples are presented to show the efficiency of our IFE spaces and methods. (Received January 22, 2014)