1110-03-230 **Megumi Sano** and **Futoshi Takahashi*** (futoshi@sci.osaka-cu.ac.jp), 3-3-138, Sugimoto, Sumiyoshi-ku, Osaka, 558-8585, Japan. *Improved Hardy inequalities in a limiting case.*

Let Ω be a bounded domain in $\mathbb{R}^N$ $(N \geq 2)$ which contains a point a and put $R = \sup_{x \in \Omega} |x-a|$. We concern the following Hardy inequalities in a limiting case:

$$\int_\Omega |\nabla u(x)|^N dx \geq \left(\frac{N-1}{N}\right)^N \int_\Omega \frac{|u(x)|^N}{|x-a|^N (\log \frac{Re}{|x-a|})^N} dx,$$

$$\int_\Omega |\nabla u(x)|^N dx \geq \left(\frac{N-1}{N}\right)^N \int_\Omega \frac{|u(x)|^N}{|x-a|^N (\log \frac{R}{|x-a|})^N} dx$$

for $u \in W_0^{1,N}(\Omega)$. It is known that the constant $\left(\frac{N-1}{N}\right)^N$ is the best one when Ω is a ball B_R and $a = 0$, and is never attained on $W_0^{1,N}(B_R)$. In this talk, we improve the above inequalities by adding nonnegative remainder terms to the right hand sides. (Received February 22, 2015)