1110-05-164 Po-Shen Loh* (ploh@cmu.edu), Wean 6113, Dept of Math Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, and Jie Ma. Diameter Critical Graphs.

A graph is called diameter-\(k\)-critical if its diameter is \(k\), and the removal of any edge strictly increases the diameter. We prove several results related to a conjecture often attributed to Murty and Simon, regarding the maximum number of edges that any diameter-\(k\)-critical graph can have. In particular, we disprove a longstanding conjecture of Caccetta and Haggkvist (that in every diameter-2-critical graph, the average edge-degree is at most the number of vertices), which promised to completely solve the extremal problem for diameter-2-critical graphs.

On the other hand, we prove that the same claim holds for all higher diameters, and is asymptotically tight, resolving the average edge-degree question in all cases except diameter-2. We also apply our techniques to prove several bounds for the original extremal question, including the correct asymptotic bound for diameter-\(k\)-critical graphs, and an upper bound of \((1/6 + o(1))n^2\) for the number of edges in a diameter-3-critical graph. (Received February 18, 2015)