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*Enumeration of lozenge tilings of a hexagon with holes on boundary.*

MacMahon's classical theorem on plane partitions fitting in a given box is equivalent to fact that the number of lozenge tilings of a centrally symmetric hexagon of side-lengths  $a, b, c, a, b, c$  (in cyclic order) on the triangular lattice is equal to

$$\frac{H(a)H(b)H(c)H(a+b+c)}{H(a+b)H(b+c)H(c+a)},$$

where the hyperfactorial function  $H(n)$  is defined by  $H(n) := 0!1!\dots(n-1)!$ .

We generalize MacMahon's theorem by giving an exact enumeration for the lozenge tilings of a hexagon with three holes on boundary. The result also solves (and generalizes) an open problem posed by James Propp (*New Perspectives in Geometric Combinatorics*, Cambridge University Press, 1999). In addition, we investigate a  $q$ -analog of the result and its connection to  $q$ -enumeration of plane partitions that fit in a connected union of several boxes. (Received February 12, 2015)