Tri Lai* (tmlai@ima.umn.edu), 207 Church Street SE, 306 Lind Hall, Minneapolis, MN 55455.

Enumeration of lozenge tilings of a hexagon with holes on boundary.

MacMahon’s classical theorem on plane partitions fitting in a given box is equivalent to the fact that the number of lozenge tilings of a centrally symmetric hexagon of side-lengths $a, b, c, a, b, c$ (in cyclic order) on the triangular lattice is equal to

$$\frac{H(a)H(b)H(c)H(a + b + c)}{H(a + b)H(b + c)H(c + a)},$$

where the hyperfactorial function $H(n)$ is defined by $H(n) := 0!1! \ldots (n − 1)!$.

We generalize MacMahon’s theorem by giving an exact enumeration for the lozenge tilings of a hexagon with three holes on boundary. The result also solves (and generalizes) an open problem posed by James Propp (New Perspectives in Geometric Combinatorics, Cambridge University Press, 1999). In addition, we investigate a $q$-analog of the result and its connection to $q$-enumeration of plane partitions that fit in a connected union of several boxes. (Received February 12, 2015)