Graph coloring and the total Betti number.

The total Betti number of the independence complex of a graph is an intriguing graph invariant. Kalai and Meshulam have raised the question on its relation to cycles and the chromatic number of a graph, and a recent conjecture on that theme was proved by Bonamy, Charbit and Thomasse. We show an upper bound on the total Betti number in terms of the number of vertex disjoint cycles in a graph. The main technique is discrete Morse theory and building poset maps.

Ramanujan graphs with arbitrary chromatic number and girth \( \log n \) is a classical construction. We show that any subgraph of them with less than \( n^{0.003} \) vertices have smaller total Betti number than some planar graph of the same order, although it is part of a graph with high chromatic number. (Received February 24, 2015)