One of the most mysterious objects associated to an elliptic curve $E$ is its Tate-Shafarevich group. Its elements can be represented by classes in the Galois-cohomology group $H^1(\mathbb{Q}, E[n])$, for various $n$. Mazur defines a class $\xi \in H^1(\mathbb{Q}, E[n])$ to be visible if there exists another elliptic curve $E'$ with $E'[n] \simeq E[n]$ such that the homogeneous space under $E'$ corresponding to $\xi$ has a rational point. Visibility provides information about the image of $\xi$ in $\Sha(E)$.

Mazur showed that any $\xi$ representing an element in $\Sha(E)[3]$ can be made visible. His proof uses that the elliptic surface obtained from the universal elliptic curve over $X(3)$ is rational. The visibility follows from the fact that a certain twist of this surface has a rational point.

The case $\xi \in H^1(\mathbb{Q}, E[4])$ is particularly interesting. The curve $X(4)$ is rational, but the relevant elliptic surface is not. It is a K3 surface. Further complications in determining the correct surface arise from the fact that 4 is even. We will discuss how to compute a model of the relevant surface given $\xi$ and give some examples of the various obstructions to rational points that can arise on these surfaces. (Received February 20, 2015)