Determining whether a system of polynomial equations has a rational solution is a long-studied problem. A first step is to determine whether the polynomial system has a solution over the real numbers and modulo N, for every positive integer N. However, this so-called local solubility is often not sufficient to guarantee the existence of a rational solution. Indeed, quadratic reciprocity and higher reciprocity laws can give nontrivial compatibility conditions among the real solutions and solutions modulo N in order for them to arise from a common rational solution. These conditions are known as the Brauer-Manin obstruction and are encoded by the Brauer group. In this talk, I will describe recent work on computing the Brauer group with a view towards these arithmetic applications. (Received February 24, 2015)