Cubic fourfolds containing a surface not homologous to a complete intersection often have nonspecial cohomology isomorphic to the primitive cohomology of a K3 surface "twisted" by an element of the Brauer group. This isomorphism is usually a manifestation of a geometric correspondence, which has consequences for the distribution of rational points on K3 surfaces over number fields. We will discuss this circle of ideas, including some recent developments in joint work with McKinnie, Sawon and Tanimoto on p-torsion Brauer classes of K3 surfaces and with Tanimoto on the Kodaira dimension of the moduli space of special cubic fourfolds of fixed discriminant. (Received February 23, 2015)