We will present some recent results on the existence of a positive solution for the following class of elliptic problems

\[-\Delta u + \lambda u = f(u), \quad \text{in } \Omega, \quad u \in H^1_0(\Omega),\]

where \( \Omega \) is an unbounded domain in \( \mathbb{R}^N \) not necessarily symmetric, \( N \geq 3 \), with smooth boundary \( \partial \Omega \neq \emptyset \) bounded, and such that \( \mathbb{R}^N \setminus \Omega \) is bounded. The non-linearity \( f \) is super-linear at zero and asymptotically linear at infinity. This result is established via a linking argument on the Nehari manifold and by means of a barycenter function. This is a work in collaboration with Benedetta Pellacci from Università degli Studi di Napoli Parthenope, Italy. (Received February 15, 2015)