Sharp singular Trudinger-Moser inequality with exact growth and its extremals

In our work, we study a sharp version of the Trudinger-Moser inequality of Adachi-Tanaka type with exact growth on $D^{1,N}(\mathbb{R}^N) \cap L^q(\mathbb{R}^N)$. We establish the inequalities with a polynomial decay and show actually we are able to get a critical equality without using the restriction of full norm. Moreover, we will see that decay allowed is the best possible. In addition, we study the existence of the extremal functions for the inequality in a subcritical case. (Received February 23, 2015)