Qi Han* (qhan@wpi.edu), Department of Mathematical Sciences, Worcester Polytechnic Institute, Worcester, MA 01609. Compact embedding results of Sobolev spaces and positive solutions to an elliptic equation in $\mathbb{R}^N$. Preliminary report.

Using a regular Borel measure $\mu \geq 0$ that vanishes on each set of capacity zero, we derive a proper subspace $D^1_{\mu}(\mathbb{R}^N)$ of $D^1(\mathbb{R}^N)$ when $N \geq 3$. This space $D^1_{\mu}(\mathbb{R}^N)$ is compactly embedded into $L^1(\mathbb{R}^N)$. An equivalence characterization and an example are provided that guarantee such a property. An example is also given if one is only interested in the compact embedding to $L^r(\mathbb{R}^N)$ for $1 \leq r < 2$. Moreover, similar results can be derived for general $D^{1,p}_{\mu}(\mathbb{R}^N)$ when $1 < p < N$, while when $p = N$, we need to use the space $W^{1,N}_{\mu}(\mathbb{R}^N)$. As an application, we prove an existence result of positive solutions to

$$-\Delta u + Vu = \lambda u^r + f(x,u)$$

in $H^1_{\mu}(\mathbb{R}^N)$ or $D^1_{\mu}(\mathbb{R}^N)$ when $N \geq 2$ without the Ambrosetti-Rabinowitz condition on $f(x,u)$. Results for the $N$-Laplacian problem can be discussed similarly when we have exponentially subcritical growth. (Received February 23, 2015)