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Sharp Moser-Trudinger Inequality on Complete Noncompact Riemannian Manifolds

We will consider the sharp Li-Ruf type critical Moser-Trudinger inequality on complete noncompact Riemannian manifolds. Namely,

$$\sup_{u \in W^{1,n}(M), \|u\|_{1,\tau} \leq 1} \int_M \phi(\alpha_n |u|^{\frac{n}{n-1}}) dV_g \leq C(n, \tau) \quad (1)$$

Where $\phi(t) = \sum_{k=n-1}^{\infty} \frac{t^k}{k!}$, α_n is the Moser's constant, $\|u\|_{1,\tau} = (\int_M \tau |u|^n + |\nabla u|^n)^{\frac{1}{n}}$. The inequality is sharp. Our method is motivated by an earlier symmetrization-free argument due to Lam and Lu in the Heisenberg group (Adv. Math. (2012)) or high order Sobolev spaces (J. Diff. Equ. (2013)) where they develop a general method to derive global Moser-Trudinger inequalities on domains of infinite measure from local ones on domains of finite measure. This method works in more general settings.

Moreover, we prove the Adachi-Tanaka's version of sharp subcritical Moser-Trudinger inequality on complete noncompact Riemannian manifolds. Namely,

$$\frac{1}{\|u\|_n^n} \int_M \phi(\alpha |u|^{\frac{n}{n-1}}) dV_g \leq C \quad (2)$$

holds for all $u \in W^{1,n}(M)$ such that $\|\nabla u\|_n \leq 1$, $\alpha < \alpha_n$. This is a joint work with Prof. Guozhen Lu. (Received February 24, 2015)