The problem of comparing the shape of two surfaces arises in many areas, including facial recognition, brain cortex analysis and computer vision. It is referred to by names such as surface registration, surface warping, best fit, shape analysis and optimal diffeomorphism. We will review some past work in this area and discuss two approaches to constructing an optimal map between a pair of surfaces. The goal is to find a good alignment of two surfaces, if they are nearly isometric, and to indicate their geometric distance if not. The first approach is based on finding an optimal conformal map between two genus-zero surfaces. A second approach is based on constructing hyperbolic orbifold metrics on surfaces and finding an energy minimizing map between them. We will then examine what these alignments reveal about the geometric similarity of shapes such as polyhedra, spheres and ellipsoids, and between shapes arising in biology, such as brain cortices, the surfaces of proteins and bones. (Received February 06, 2015)