The dual Ramsey’s theorem (DRT) states that given a nice finite coloring of all partitions of $\omega$ into $k$ parts, there is a partition of $\omega$ into infinitely many parts, every coarsening of which into $k$ parts has the same color. The niceness condition on the coloring here is a topological one, usually having the property of Baire or being Borel. However, most work on DRT in computability theory has focused on the case where the colorings are topologically open. We show that, in the case of colorings with the property of Baire, the latter is actually not a restriction as far as logical strength is concerned, as the two versions are equivalent over $RCA_0$. By contrast, we show that the Borel formulation is only equivalent to the open one over $ATR_0$. While the Borel DRT implies $ATR_0$, it is unknown whether the Baire DRT does as well, so by the preceding result this question is equivalent to whether or not the Borel and Baire versions of DRT are equivalent.

We also identify several combinatorial variants and fragments of the Borel DRT, and study their relationships to the Carlson-Simpson lemma, and to the stable Ramsey’s theorem for pairs. This is joint work with Stephen Flood, Reed Solomon, and Linda Brown Westrick. (Received August 11, 2015)