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**Rutger Kuyper\*** ([mail@rutgerkuyper.com](mailto:mail@rutgerkuyper.com)), Department of Mathematics, University of Wisconsin, 480 Lincoln Dr., Madison, WI 53706-1388. *A measure of uniformity.*

There are two well-known ways to compare the difficulty of two *mass problems*, i.e. of sets  $\mathcal{A}, \mathcal{B} \subseteq \omega^\omega$ . The first of these, *Medvedev reducibility*, says that  $\mathcal{A}$  reduces to  $\mathcal{B}$  if there is a single Turing functional  $\Phi$  such that  $\Phi(\mathcal{B}) \subseteq \mathcal{A}$ . On the other hand, we say that  $\mathcal{A}$  *Muchnik reduces* to  $\mathcal{B}$  if every element of  $\mathcal{B}$  computes an element of  $\mathcal{A}$ .

These two reducibilities can be seen as two opposite extremes: Medvedev reducibility is as uniform as possible, while Muchnik reducibility is as non-uniform as possible. Therefore, if some reduction is not completely uniform, one can only conclude that it is a Muchnik reduction, even though intuitively it might feel like there is some uniformity contained in the reduction.

We propose a hierarchy of reducibilities between Medvedev and Muchnik reducibility, which capture different levels of uniformity. We say that  $\mathcal{A}$  *n-uniformly reduces* to  $\mathcal{B}$  if there is a  $\Pi_n^0$ -cover  $\mathcal{V}_0, \mathcal{V}_1, \dots$  of  $\mathcal{B}$  such that the elements from  $\mathcal{B} \cap \mathcal{V}_i$  uniformly compute elements from  $\mathcal{A}$ . We use this concept to measure the uniformity of some well-known Muchnik reductions. (Received August 11, 2015)