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Jinyu Huang (jhuang14@hawk.iit.edu) and **Hemanshu Kaul*** (kaul@iit.edu), Illinois Institute of Technology, 10 W 32nd St., Chicago, IL 60616. *On Matroid Expansion Conjecture: Counting Bases of a Matroid.*

Matroid Bases generalize many fundamental combinatorial structures. It is a longstanding problem to design an efficient algorithm for counting the number of bases in a matroid. A natural Markov Chain Monte Carlo algorithm would sample bases from the base-exchange graph G of the matroid M . The vertex-set of G is the collection of all bases of M and two bases are adjacent in G if their symmetric difference is exactly two elements. It was conjectured by Mihail and Vazirani in 1989 that the cutset-expansion (or conductance) of any base-exchange graph is at least 1, which is called the Matroid-expansion conjecture. If this conjecture is true then the natural MCMC algorithm is in fact rapidly convergent and defines an FPRAS (fully-polynomial randomized approximation scheme) for efficiently counting the number of bases of a matroid.

We give polynomial (or constant) bounds on the second smallest eigenvalue, λ_2 , of the discrete Laplacian of the Base-exchange graph (which implies a bound on its conductance) for base-transitive matroids, paving matroids, and balanced matroids. This implies that the Matroid expansion conjecture is true for paving matroids, balanced matroids, and their direct sums; extending the results of Feder and Mihail (1992) and Jerrum (2006). (Received August 11, 2015)