Matroid Bases generalize many fundamental combinatorial structures. It is a longstanding problem to design an efficient algorithm for counting the number of bases in a matroid. A natural Markov Chain Monte Carlo algorithm would sample bases from the base-exchange graph $G$ of the matroid $M$. The vertex-set of $G$ is the collection of all bases of $M$ and two bases are adjacent in $G$ if their symmetric difference is exactly two elements. It was conjectured by Mihail and Vazirani in 1989 that the cutset-expansion (or conductance) of any base-exchange graph is at least 1, which is called the Matroid-expansion conjecture. If this conjecture is true then the natural MCMC algorithm is in fact rapidly convergent and defines an FPRAS (fully-polynomial randomized approximation scheme) for efficiently counting the number of bases of a matroid.

We give polynomial (or constant) bounds on the second smallest eigenvalue, $\lambda_2$, of the discrete Laplacian of the Base-exchange graph (which implies a bound on its conductance) for base-transitive matroids, paving matroids, and balanced matroids. This implies that the Matroid expansion conjecture is true for paving matroids, balanced matroids, and their direct sums; extending the results of Feder and Mihail (1992) and Jerrum (2006). (Received August 11, 2015)