Denote by $K_p(n, k)$ the random subgraph of the usual Kneser graph $K(n, k)$ in which edges appear independently, each with probability $p$. Answering a question of Bollobás, Narayanan, and Raigorodskii, we show that there is a fixed $p < 1$ such that a.s. (i.e., with probability tending to 1 as $k \to \infty$) the maximum independent sets of $K_p(2k + 1, k)$ are precisely the sets $\{A \in V(K(2k + 1, k)) : x \in A\}$ (where $x \in [2k + 1]$). We also complete the determination of the order of magnitude of the “threshold” for the above property for general $k$ and $n \geq 2k + 2$. This is new for $k \sim n/2$, while for smaller $k$ it is a recent result of Das and Tran. Joint work with Jeff Kahn. (Received August 11, 2015)