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The facets of the cut polytope and the extreme rays of cone of concentration matrices of certain graphs. Preliminary report.

For a graph G with p vertices the cone of concentration matrices consists of all real positive semidefinite $p \times p$ matrices with zeros in entries corresponding to nonedges of G . The extremal rays of this cone and their associated ranks are well-studied with applications in matrix completion problems, maximum likelihood estimation, and Gauss elimination of sparse matrices. It is well-known that the extremal rays of this cone in the case of the cycle are either rank 1 or rank $p - 2$. Similarly, the cut polytope of the cycle has facets of two distinct shapes. With hyperplane translations and general duality theory of spectrahedra, we demonstrate that a facet of a fixed shape corresponds to an extremal ray of a fixed rank. This shows that, in the case of the cycle, the different facet shapes in the cut polytope identify the ranks of extremal rays in the cone of concentration matrices, and this correspondence arises from the cutsets defining the facets. More generally, we show that if a graph G has this property, then taking the clique sum of G with either a cycle or a tree produces another graph with the same property. (Received March 29, 2015)