Let $P$ be a nonempty set and $\Gamma \subseteq \text{Sym } P$ be a regular permutation set acting on $P$. It is well known that, upon fixing $o \in P$, we can associate to $(P, \Gamma, o)$ a loop $(P, +)$ and conversely to any loop we can associate a suitable regular permutation set. The detailed study of the relationships between these structures is used here for building a new loop $(L, \oplus)$ starting from the loops $(K, +)$, equipped with a well ordering “$\preceq$”, $(P, \hat{+})$ and assuming that a further loop operation “$+$” (which may also coincide with “$\hat{+}$”) is defined on $P$.

We call the loop $(L, \oplus)$ slid extension of $P$ by $K$. We study the dependence of the properties of the new loop $(L, \oplus)$ on the corresponding properties of the initial ones (associativity, automorphic inverse properties, Bol and Moufang conditions) and characterize the nuclei of $(L, \oplus)$. Most of the results presented here appeared in [1].

This procedure can provide examples of proper loops also when the initial loops are groups.

References


(Received July 23, 2015)