Supercharacter theories have given us new ways to find the representation theory underlying Hopf structures. One of the most striking examples is the Hopf algebra of symmetric functions in noncommuting variables arising from a Hopf monoid in the representation theory of unipotent groups. In this case, we obtain Hopf structures built on set partitions. It turns out that set partitions may be viewed as integer lattice points contained in a family of transportation polytopes. This talk describes a supercharacter theory for a family of groups that is built on the lattice points of arbitrary transportation polytopes giving rise to a corresponding set of Hopf structures. (Received August 04, 2015)