1112-35-74 **Rafael D. Benguria** (rbenguri@fis.puc.cl) and **Soledad Benguria*** (benguria@math.wisc.edu). *The Brezis–Nirenberg Problem on $\mathbb{S}^n$, in spaces of fractional dimension.*

We consider the nonlinear eigenvalue problem,

$$-\Delta_{\mathbb{S}^n} u = \lambda u + |u|^{4/(n-2)} u,$$

with $u \in H_0^1(\Omega)$, where Ω is a geodesic ball of radius θ_1 in $\mathbb{S}^n$. In dimension $n = 3$, Bandle and R. Benguria proved that if $\lambda \geq -n(n-2)/4$ this problem has a unique positive solution if and only if $\lambda \in ((\pi^2 - 4\theta_1^2)/4\theta_1^2, \lambda_1)$, where λ_1 is the first Dirichlet eigenvalue. For positive radial solutions of this problem one is led to an ordinary differential equation that still makes sense when n is a real rather than a natural number. Here we consider precisely that situation with $2 < n < 4$. Our main result is that in this case one has a positive solution if and only if $\lambda \geq -n(n-2)/4$ is such that

$$\frac{1}{4}[(2\ell^* + 1)^2 - (n-1)^2] < \lambda < \lambda_1$$

where ℓ^* is the first positive value of ℓ for which the associated Legendre function $\mathrm{P}_\ell^{(n-2)/2}(\cos\theta_1)$ vanishes. (Received July 15, 2015)