Rafael D. Benguria (rbenguri@fis.puc.cl) and Soledad Benguria*
(benguria@math.wisc.edu). The Brezis–Nirenberg Problem on \( S^n \), in spaces of fractional dimension.

We consider the nonlinear eigenvalue problem,

\[-\Delta_{S^n} u = \lambda u + |u|^{4/(n-2)}u,\]

with \( u \in H^1_0(\Omega) \), where \( \Omega \) is a geodesic ball of radius \( \theta_1 \) in \( S^n \). In dimension \( n = 3 \), Bandle and R. Benguria proved that if \( \lambda \geq -n(n-2)/4 \) this problem has a unique positive solution if and only if \( \lambda \in ((\pi^2 - 4\theta_1^2)/4\theta_1^2, \lambda_1) \), where \( \lambda_1 \) is the first Dirichlet eigenvalue. For positive radial solutions of this problem one is led to an ordinary differential equation that still makes sense when \( n \) is a real rather than a natural number. Here we consider precisely that situation with \( 2 < n < 4 \).

Our main result is that in this case one has a positive solution if and only if \( \lambda \geq -n(n-2)/4 \) is such that

\[
\frac{1}{4}[(2\ell^* + 1)^2 - (n-1)^2] < \lambda < \lambda_1
\]

where \( \ell^* \) is the first positive value of \( \ell \) for which the associated Legendre function \( P_{\ell}^{(n-2)/2}(\cos \theta_1) \) vanishes. (Received July 15, 2015)