1098-05-311 **James Allen Fill***, jimfill@jhu.edu, and **Alan D. Sokal**. *The leading root of a formal power series $f(x,y) = \sum_{n=0}^{\infty} a_n(y)\, x^n$, with connections to probability.* Preliminary report.

This talk will concern the "leading root" (unique power-series root) $x_0(y)$ of a formal power series $f(x,y) = \sum_{n=0}^{\infty} a_n(y)\, x^n$, where the series $a_n(y)$ have nonzero constant terms for $n = 0, 1$ and for $n \geq 2$ satisfy modest smallness conditions such as $a_n(y) = O(y^{\alpha(n-1)})$ for some $\alpha > 0$. Problems of this type arise frequently in combinatorics, statistical mechanics, number theory, and analysis.

A prominent example is the "deformed exponential function" (DE)

$$f(x,y) = \sum_{n=0}^{\infty} \frac{y^{n(n-1)/2}}{n!} x^n.$$

For this example, let $U(y) := -x_0(y)$. Extensive numerical computations lead to the conjecture that $U(y)$ has all strictly positive coefficients; and, even more strongly, that $F(y) := 1 - [1/U(y)]$ has all strictly positive coefficients after the vanishing constant term. This is just the proverbial tip of the iceberg, in that similar positivity properties, both for the leading root and for approximations to the leading root used for efficient computation of its coefficients, are conjectured for wide families of examples that include DE.

There are almost no proofs available yet for these conjectures, but I will discuss exceptions. I will also discuss several connections of these problems with probability. (Received January 28, 2014)