Non-rigid parabolic geometries of Monge type.

A parabolic geometry, with the classical examples of conformal geometry, projective geometry and CR-geometry, is defined by a choice of a parabolic subalgebra in a simple Lie algebra. In particular, the geometry of the (2, 3, 5) distributions on a 5-manifold is the parabolic geometry associated to the simple Lie algebra $\mathfrak{g}_2$ with the parabolic subalgebra defined by the first simple root. Associated with the flat model for this geometry there is a natural underdetermined ODE, the celebrated Hilbert-Cartan equation $\frac{dz}{dx} = \left(\frac{d^2y}{dx^2}\right)^2$. In this talk, we will generalize the above example of parabolic geometry to all simple Lie algebras in the framework of non-rigid parabolic geometry of Monge type. The differential equations for the corresponding flat models are all underdetermined systems of ODE’s. A parabolic geometry is called non-rigid if it allows curved analogs, and the non-rigidity is characterized by the second Lie algebra cohomology. For the non-rigid parabolic geometries of Monge type, the ODE systems have particularly simple forms. We will also mention work in progress on the underlying geometric structures to define the curved analogs of these geometries. (Received January 28, 2014)