Our starting point is the heat flow operator which is used to describe a number of physical phenomena. Recall that its generator is the Laplace operator. In particular, a well-known connection between the spectrum of the Laplacian and the speed of heat diffusion leads to several functional inequalities such as Poincare, Nash etc. The geometry of the underlying space plays an important role in such an analysis. Another example of a functional inequality is the log-Sobolev inequality which is used to describe entropic convergence of the heat flow to an equilibrium. A probabilistic point of view comes from a path integral representation of the heat flow for stochastic differential equations driven by a Brownian motion. The talk will review recent advances in the field including elliptic and hypo-elliptic settings over both finite- and infinite-dimensional spaces. (Received January 22, 2014)