Feynman-Kac formulae for solutions to degenerate elliptic boundary value and obstacle problems with Dirichlet boundary conditions.

We prove stochastic representation formulae for solutions to the elliptic boundary value and obstacle problems associated with a degenerate Markov diffusion process. The degeneracy in the diffusion coefficient is proportional to the $\alpha$-power of the distance to the boundary of the half-space, where $\alpha \in (0, 1)$. This generalizes the well-known Heston stochastic volatility process, which is widely used as an asset price model in mathematical finance and a paradigm for a degenerate diffusion process. The generator of this degenerate diffusion process with killing, is a second-order, degenerate-elliptic partial differential operator where the degeneracy in the operator symbol is proportional to the $2\alpha$-power of the distance to the boundary of the half-plane. Our stochastic representation formulae provide the unique solutions to the elliptic boundary value and obstacle problems, when we seek solutions which are suitably smooth up to the boundary portion $\Gamma_0$ contained in the boundary of the half-plane. In the case when the full Dirichlet condition is given, our stochastic representation formulae provide the unique solutions which are not guaranteed to be any more than continuous up to the boundary portion $\Gamma_0$. (Received January 13, 2014)