Fix a prime $p > 2$. Let $\rho$ be the Galois representation coming from a non-CM irreducible component $\mathcal{I}$ of Hida’s $p$-ordinary Hecke algebra. Assume the residual representation $\bar{\rho}$ is absolutely irreducible. Under a minor technical condition, we identify a subring $\mathcal{I}_0$ of $\mathcal{I}$ containing $\mathbb{Z}_p[[T]]$ such that the image of $\rho$ is large with respect to $\mathcal{I}_0$. That is, the image of $\rho$ contains $\ker(\text{SL}_2(\mathcal{I}_0) \to \text{SL}_2(\mathcal{I}_0/a))$ for some non-zero $\mathcal{I}_0$-ideal $a$. This paper builds on recent work of Hida who showed that the image of such a Galois representation is large with respect to $\mathbb{Z}_p[[T]]$. Our result is an $\mathcal{I}$-adic analogue of the description of the image of the Galois representation attached to a non-CM classical modular form obtained by Ribet and Momose in the 1980s. (Received August 26, 2015)