We present joint work with Bruce Jordan and Anthony Scholl on the Jacobian $J$ of the modular curve $X = X_0(N)$, where $N$ is a positive integer.

Let $\tilde{T}$ be the ring of Hecke operators on the space of modular forms of weight 2 for $\Gamma_0(N)$, and let $T$ be the image of $\tilde{T}$ in the endomorphism ring of $J$. The Eisenstein ideal of $T$ is the ideal of those $t \in T$ that lift to an operator $\tilde{t} \in \tilde{T}$ such that $\tilde{t}$ vanishes on the space of Eisenstein series.

Let $C$ be the cuspidal subgroup of $J$. Because we have $I \subseteq \text{Ann}_T C$, it is natural to ask whether $I = \text{Ann}_T C$.

We prove this equality locally at prime numbers that are prime to the product $6N$ and expect to be able to consider more generally primes (including 2 and 3) whose squares do not divide $N$.

Let $C$ be the formal cuspidal group for $J$, the group of degree-0 divisors on $X$ with support on the cusps. There is a natural map $C \to J$ whose image is $C$; we regard this map as a 1-motive $[C \to J]$. Consideration of the cohomology of this 1-motive reveals the desired connection between the Eisenstein ideal and the cuspidal group of $J$. (Received September 01, 2015)