In this talk, we consider symmetries of spatial graphs in closed orientable 3-manifolds described by smooth finite group actions. We first show that there is an infinite family of hyperbolic spatial graphs with given symmetry. Next, we apply this method to the study of links in 3-manifolds which can be regarded as systems of rotation axes in closed hyperbolic 3-manifolds obtained by Dehn surgeries. (Received August 30, 2015)