The mechanical system of one rigid surface rolling over another without twisting or slipping is a staple of non-holonomic mechanics and has been studied from a number of different points of view. The differential equations that describe this motion turn out to be a special case of a system of PDE studied by Élie Cartan in 1910. Remarkably, Cartan showed that such systems can have a symmetry group with dimension as large as 14 (and that, in this case, the symmetry group is isomorphic to the exceptional group $G_2$). For example, it turns out that a sphere of radius 1 rolling over a sphere of radius 3 belongs to this highly symmetric case.

In recent years, there have been some surprising developments; Nurowski and An have discovered a remarkable convex surface in 3-space whose differential constraints that describe its rolling over the flat plane have $G_2$-symmetry.

In this talk, I will describe the history of this problem, the geometry that goes into its study, and the recent developments, including some results of my own that provide progress in classifying the pairs of surfaces whose rolling constraints have exceptional symmetry. (Received August 27, 2014)