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Sébastien Labbé* (labbe@liafa.univ-paris-diderot.fr) and **Christophe Reutenauer**. *A d -dimensional extension of Christoffel words.*

In this article, we extend the definition of Christoffel words to directed subgraphs of the hypercubic lattice in arbitrary dimension that we call Christoffel graphs. Christoffel graphs when $d = 2$ correspond to well-known Christoffel words. Due to periodicity, the d -dimensional Christoffel graph can be embedded in a $(d - 1)$ -torus (a parallelogram when $d = 3$). We show that Christoffel graphs have similar properties to those of Christoffel words: symmetry of their central part and conjugation with their reversal. Our main result extends Pirillo's theorem (characterization of Christoffel words which asserts that a word amb is a Christoffel word if and only if it is conjugate to bma) in arbitrary dimension. In the generalization, the map $amb \mapsto bma$ is seen as a flip operation on graphs embedded in \mathbb{Z}^d and the conjugation is a translation. We show that a fully periodic subgraph of the hypercubic lattice is a translate of its flip if and only if it is a Christoffel graph. (Received August 18, 2014)