

1103-47-7

Steven H. Weintraub* (shw2@lehigh.edu), Lehigh University, Bethlehem, PA 18015. *The adjoint of differentiation.*

Let n be any nonnegative integer. Let $V = P_n$ be the vector space of polynomials of degree at most n , equipped with the inner product $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$. Let $D : V \rightarrow V$ be the differentiation operator, $D(f) = f'$. Then D has an adjoint D^* . We have closed form expressions for D^* , which were conjectured by computing D^* for small values of n and finding a pattern. (If $f(x)$ is a polynomial of degree $k \leq n$, then, while the value of $D(f(x))$ is independent of n , the value of $D^*(f(x))$ depends on n .) We also find formulas for D^* in terms of classical Legendre polynomials, shifted to the interval $[0, 1]$. Using these formulas it is easy to prove that our closed form expressions are correct. An alternate approach yields combinatorial identities involving the entries of the inverses of Hilbert matrices. (Received May 22, 2014)