

1109-37-301

Francesco Cellarosi* (fcellaro@illinois.edu), 1409 W Green Street, URBANA, IL 61801.

Quadratic Weyl sums, Automorphic Functions, and Invariance Principles.

In 1914, Hardy and Littlewood published their celebrated approximate functional equation for quadratic Weyl sums. Their result provides, by iterative application, a powerful tool for the asymptotic analysis of such sums. We construct a related, almost everywhere non-differentiable automorphic function, which approximates quadratic Weyl sums up to an error of order one, uniformly in the summation range. This not only implies the approximate functional equation, but allows us to replace Hardy and Littlewood's renormalization approach by the dynamics of a certain homogeneous flow. The great advantage of this construction is that the approximation is global, i.e., there is no need to keep track of the error terms accumulating in an iterative procedure. Our main application is a new functional limit theorem, or invariance principle, for theta sums. The interesting observation is that the paths of the limiting process share a number of key features with Brownian motion (scale invariance, invariance under time inversion, non-differentiability), although time increments are not independent, the value distribution at each fixed time is distinctly different from a normal distribution. Joint work with Jens Marklof. (Received February 03, 2015)