We study the existence and uniqueness of the stationary distribution of an infinite-dimensional Markov process that arises as the diffusion limit of many-server queues. This process consists of two coupled components, the first component being a real-valued Itô process $X$ with a constant diffusion coefficient and the second being a process $Z$ that takes values in a subset of the Hilbert function space $\mathbb{H}^1$ and satisfies a somewhat unusual stochastic partial differential equation (SPDE) with a boundary condition that depends on $X$. Standard Harris recurrence methods for studying uniqueness of stationary distributions are not applicable here because they are not well suited to establishing uniqueness of stationary distributions for infinite-dimensional Markov processes. Instead, we use an asymptotic coupling approach to establish uniqueness, thus demonstrating the applicability of this method in the context of non-standard SPDEs that arise in the analysis of queueing systems. This is joint work with Mohammadreza Aghajani. (Received January 28, 2015)