We show that for any finite coloring of the Gaussian integers, there exist distinct nonzero elements $x$ and $y$ of the same color such that $x^2 - y^2 = n^2$ for some Gaussian integer $n$. We will also explain the connection between this problem with a structure theorem for multiplicative functions on the Gaussian integers. (Received January 11, 2015)