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Paul C. Kainen* (kainen@georgetown.edu). *Cogenus-1-drawings of complete bipartite graphs.*

In 1965, Ringel determined the genus γ_r of $K_{r,r}$, $\gamma_r = (1/4)(r-2)^2$. We show how to extend Zarankiewicz's plane drawing of $K_{r,r}$ to a drawing on the orientable surface of genus $-1 + \gamma_r$ with exactly r crossings, provided that 4 divides r and $r \geq 8$. E.g., $K_{8,8}$ has crossing number at most 8 on the surface obtained from the sphere by attaching 8 handles. The construction involves attaching handles to the plane (or sphere) such that handles carry the edges of a $K_{2,2}$ subgraph. Our method is incremental, building the representation of $K_{r+4,r+4}$ from that of $K_{r,r}$. Furthermore, there is a vertex-disjoint covering of the vertices by crossing-free quadrilaterals which bound disks, so one can obtain low crossing number drawings of $K_{r,r} \times Q_d$ as in Kainen and White (J. Graph Theory, 1978). Other advantages of the new drawing scheme are that it is easier to visualize and can be adapted to deal with irregular graphs. (Received January 21, 2015)