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**Marcus G Schaefer\*** (mschaefer@cdm.depaul.edu), Chicago, IL 60604, and **Daniel Stefankovic**, Rochester, NY 14627. *The Degenerate Crossing Number and Higher-Genus Embeddings.*

Suppose a graph can be embedded in a surface with  $k$  crosscaps. Is there always an embedding (in the same surface) in which every edge passes through each crosscap at most once? This well-known open problem can be restated using crossing numbers: the degenerate crossing number,  $\text{dcr}(G)$  of  $G$  equals the smallest number  $k$  so that  $G$  has an embedding in a surface with  $k$  crosscaps in which every edge passes through each crosscap at most once. The genus crossing number,  $\text{gcr}(G)$  of  $G$  equals the smallest number  $K$  so that  $G$  has an embedding in a surface with  $k$  crosscaps. The original question then asks whether  $\text{dcr}(G) = \text{gcr}(G)$ .

We show that  $\text{dcr}(G) \leq 6 \text{gcr}(G)$ , and  $\text{dcr}(G) = \text{gcr}(G)$  as long as  $\text{dcr}(G) \leq 3$ . We can separate  $\text{dcr}$  and  $\text{gcr}$  for (single-vertex) graphs with embedding schemes, but it is not clear whether the separating example can be extended into separations on simple graphs. Finally, we show that if a graph can be embedded in a surface with crosscaps, then it has an embedding in that surface in which every edge passes through each crosscap at most twice. (Received November 17, 2014)