The asymptotic patterns of the Betti sequences of the finitely generated modules over a local ring $R$ reflect and affect the singularity of $R$. For instance, these sequences are eventually zero if and only if $R$ is regular (Auslander and Buchsbaum, Serre) and they are eventually constant if and only if $R$ is a hypersurface (Shamash, Gulliksen, Eisenbud). We describe those rings over which the next simplest pattern occurs—each Betti sequences is eventually arithmetic. More generally, when $c$ is a non-negative integer we obtain sufficient conditions and necessary conditions for each Betti sequences to be eventually given by some polynomial of degree less than $c$. These conditions coincide when $c \leq 3$ or when $R$ is homogeneous. (Received January 14, 2015)