Test ideals measure singularities of schemes in characteristic $p$ by quantifying the failure of the flatness of Frobenius. Surprisingly, they are closely tied with multiplier ideals in characteristic zero which are defined by completely different means. Like multiplier ideals, the theory of test ideals is much better understood when the ambient ring is $\mathbb{Q}$-Gorenstein. In this paper, we show that if the $\mathbb{Q}$-Gorenstein hypothesis is replaced with the hypothesis that the anticanonical section ring is finitely generated, then many results still hold (note that this condition is expected to hold in every strongly $F$-regular ambient ring). In particular, we obtain discreteness and rationality of $F$-jumping numbers, we obtain global generation results for test ideals, we show that multiplier ideals turn into test ideals under reduction mod $p$, we obtain descriptions of test ideals via alterations, and in particular we reprove as a special case an unpublished result of Singh showing that splinters are strongly $F$-regular in this setting. This is joint work with Alberto Chiecchio, Florian Enescu, and Lance Miller. (Received January 17, 2015)