Let $F$ be a field, let $D$ be a subring of $F$ and let $\mathcal{X}$ denote the set of valuation rings having quotient field $F$. Then $\mathcal{X}$ admits the Zariski, inverse and patch topologies, all of which are spectral and have been well studied in recent years. This point of view can be further enriched by viewing $\mathcal{X}$ as a ringed space with respect to appropriate sheaves of $D$-algebras for each of these topologies. Viewing $\mathcal{X}$ with an appropriate locally ringed space structure and considering morphisms into the projective line then makes it possible to distinguish properties of intersection of valuation rings (i.e., integrally closed rings) such as whether the intersection is Prüfer or irredundant. Other sheaf structures shed additional light on these intersections. We discuss some aspects of these constructions and consider them also in the context of star operations. (Received January 20, 2015)