1107-52-179 **Gabor Toth*** (gtoth@camden.rutgers.edu), Rutgers University, Department of Mathematics, Camden, NJ 08102, and **Qi Guo**, Suzhou University of Science and Technology, Department of Mathematics, Suzhou, Jiangsu 215009, Peoples Rep of China. *Dual Mean Minkowski Measures and the Grünbaum Conjecture.* Preliminary report.

For a convex body $C \subset \mathbb{R}^n$, we define two sequences $\{\sigma_{C,k}\}_{k\geq 1}$ and $\{\sigma^o_{C,k}\}_{k\geq 1}$ of functions on the interior of C. The k-th members are "mean Minkowski measures in dimension k" which are pointwise dual: $\sigma^o_{C,k}(O) = \sigma_{C^o,k}(O)$, where $O \in \operatorname{int} C$, and C^o is the dual of C with respect to O. We have

$$1 \leq \sigma_{C,k}(O), \sigma^o_{C,k}(O) \leq \frac{k+1}{2}.$$

The lower bound is attained iff C has a k-dimensional simplicial slice or simplicial projection. The upper bound is attained iff C is symmetric with respect to O. Klee showed that the condition $m^*_C > n-1$ on the Minkowski measure of C implies that there are $n+1$ affinely independent affine diagonals meeting at a critical point $O^* \in C$. In 1963 Grünbaum conjectured the existence of such point in any convex body. While this conjecture remains open (and difficult), as a byproduct of the properties of the dual mean Minkowski measures, we show that

$$\frac{n}{m^*_C + 1} \leq \sigma_{C,n-1}(O^*),$$

and if sharp inequality holds then the Grünbaum conjecture holds. Our assumption is much weaker than Klee's. (Received January 13, 2015)