Recently, the authors improved the bound on the largest gaps between consecutive prime numbers. An important ingredient in the proof is a new result on efficient covering of hypergraphs, which extends a well-known theorem of Pippenger and Spencer from 1989. The Pippenger-Spencer theorem ensures the existence of a near-perfect packing of a hypergraph $H = (V, E)$ under three basic assumptions on $H$: (a) uniformity - all hyperedges $e \in E$ have the same (bounded) cardinality $k$; (b) regularity - the degree of each vertex $v \in V$ is asymptotically the same; (c) small codegrees - for all distinct $v, w \in V$, there are "few" edges containing both $v$ and $w$. Our new theorem gives essentially the same conclusion (not necessarily a packing, but an efficient near-covering) with a substantial weakening of hypotheses (a) and (b), while retaining (c) and the main hypothesis. (Received July 09, 2015)