Given a labeled graph $G$ with adjacency matrix $A$, we define the adjacency algebra of $G$ to be the matrix algebra $\mathcal{A}$ generated by $A$. Since $A$ is diagonalizable, the dimension of $\mathcal{A}$ is simply the number of distinct eigenvalues of $A$. If $G$ is connected and this algebra is also closed under the Schur (entrywise) product of matrices, we will call the graph $G$ an $S$-graph.

Distance-regular graphs are examples of $S$-graphs, however many other interesting graphs which are not distance-regular fall into this class as well. This talk will begin with a combinatorial characterization of $S$-graphs followed by examples. We will then discuss the ongoing search for examples of certain types of $S$-graphs, including those with the so-called $Q$-polynomial property. These are $S$-graphs whose idempotents $E_0, E_1, \ldots, E_d$ in the spectral decomposition of $A$ can be ordered so that $E_i$ is a degree $i$ polynomial of $E_1$, where multiplication is the Schur product. (Received August 25, 2015)