Let $X$ be a separable Hilbert space and let the linear operator $A$ generate a $C_0$-semigroup on $X$. Within the framework of linear control theory, the observation problem on a finite time horizon $T > 0$ typically reads as

$$
\dot{z}(t) = Az(t) \quad \text{for} \quad t \in (0, T), \quad z(0) = x,
$$

$$
w(t) = Cz(t) \quad \text{in} \quad (0, T)
$$

for some observation operator $C$.

In this talk, we assume the observation variable $w$ to be 1D and consider a noisy system given by

$$
\dot{z}(t) = Az(t) \quad \text{for} \quad t \in (0, T), \quad z(0) = x,
$$

$$
w(t_k) = Cz(t_k) + \varepsilon(t_k) \quad \text{for} \quad k = 1, \ldots, n,
$$

where $t_k = Tk/n$ and $\varepsilon(t_k)$’s are i.i.d. univariate r.v. with mean 0 and variance $\sigma^2 > 0$. Note that the system is now observed over a discrete set of time periods.

Assuming the deterministic system is exactly observable at time $T$, we use the taut string estimator from nonparametric statistics to construct an estimate $\hat{x}_n$ for the initial state $x$ based on noisy observations and prove $\hat{x}_n$ converges in appropriate sense to the actual initial state $x$ reconstructed from the original deterministic system at the optimal rate of $n^{-1/2}$. (Received August 21, 2015)