In 1980 J. Bourgain and F. Delbaen introduced a construction method, used to obtain $L_\infty$-spaces not containing $c_0$. A large variety of $L_\infty$-spaces has been constructed with this method, such as an example is the Argyros-Haydon space, the first Banach space satisfying the scalar-plus-compact property. Based on the aforementioned construction, we give a general definition of a Bourgain-Delbaen space and prove that every separable $L_\infty$-space is isomorphic to such a space. We use this general approach to obtain Bourgain-Delbaen spaces as quotients of simpler Bourgain-Delbaen spaces. This is analogous to the use of an unconditional norming set as the frame for an HI construction. We also mention some recent examples of $L_\infty$-spaces, such as an asymptotic $c_0$ $L_\infty$-space not containing $c_0$ and a space with the scalar-plus-compact property having no reflexive subspaces.

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