Inspired by ideas of Schatten in his celebrated monograph [A theory of cross-spaces, 1950], we introduce the notion of a Lipschitz tensor product $X \boxtimes E$ of a pointed metric space $X$ and a Banach space $E$ as a certain linear subspace of the algebraic dual of $\text{Lip}_0(X, E^*)$, where $\text{Lip}_0(X, E^*)$ denotes the space of Lipschitz functions from $X$ to $E^*$.

We show that the Lipschitz injective norm $\varepsilon$, the Lipschitz projective norm $\pi$ and the Lipschitz $p$-nuclear norm $d_p$ ($1 \leq p \leq \infty$) are uniform dualizable Lipschitz cross-norms on $X \boxtimes E$ and study their properties.

On the other hand, for a Lipschitz cross-norm $\alpha$ on $X \boxtimes E$, we introduce the notion of $\alpha$-Lipschitz operators from $X$ into $E^*$ and prove that the space $\text{Lip}_\alpha(X, E^*)$ of such Lipschitz operators under the $\alpha$-Lipschitz norm $\text{Lip}_\alpha$ is isometrically isomorphic to the dual space of $X \boxtimes_\alpha E$.

In addition, if $p'$ denotes the conjugate index of $p$, we show that $\text{Lip}_{d_p}(X, E^*)$ is justly the space of all Lipschitz $p'$-summing operators from $X$ to $E^*$ (introduced by Farmer and Johnson) and therefore such space can be identified with $(X \boxtimes_{d_p} E)^*$. (Received August 17, 2015)