As the Great Financial Crisis reminds us, extreme movements in the level and volatility of asset prices are key features of financial markets. These phenomena are difficult to quantify using traditional approaches that specify extreme risk as a singular rare event detached from ordinary dynamics. Multifractal analysis, whose use in finance has considerably expanded over the past fifteen years, reveals that price series observed at different time horizons exhibit several major forms of scale-invariance. Building on these regularities, researchers have developed a new class of multifractal processes that permit the extrapolation from high-frequency to low-frequency events and generate accurate forecasts of asset volatility. The new models provide a structured framework for studying the likely size and price impact of events that are more extreme than the ones historically observed. (Received August 15, 2015)