Jonathan Sondow* (jsondow@alumni.princeton.edu) and Kieren MacMillan. Primary Pseudoperfect Numbers, Arithmetic Progressions, and the Erdős-Moser Equation.

A primary pseudoperfect number (PPN for short) is an integer $K > 1$ that satisfies the equation

$$\frac{1}{K} + \sum_{p|K} \frac{1}{p} = 1,$$

where $p$ denotes a prime. PPNs arise in studying perfectly weighted graphs and singularities of algebraic surfaces, and are related to Sylvester's sequence and Curtiss's bound on solutions to a unit fraction equation.

For any PPN $K$ we show that $K \equiv 6 \pmod{6^2}$ if $6 \mid K$, and we uncover a remarkable 7-term arithmetic progression of residues modulo $6^2 \cdot 8$ in the sequence 6, 42, 1806, 47058, 2214502422, 52495396602, 849042158359688410706771261086 of known PPNs $K > 2$. On that basis, we pose a conjecture which leads to a conditional proof of the new record lower bound $k > 10^{3.99 \times 10^{20}}$ on any non-trivial solution to the Erdős-Moser Diophantine equation $1^n + 2^n + \cdots + k^n = (k + 1)^n$. Assuming the Riemann Hypothesis, we obtain the slightly better, but doubly conditional, bound $k > 10^{4 \times 10^{20}}$.

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