The classical theory of spherical harmonics on $\mathbb{R}^3$ was generalized by Kostant in his 1963 paper *Lie group representations on polynomial rings*, which in turn was generalized by Kostant and Rallis in their 1971 paper *Orbits and representations associated with symmetric spaces*. Much of the theory in the latter was generalized by Vinberg’s 1976 theory of $\theta$-groups in *The Weyl group of a graded Lie algebra*. A survey of Vinberg’s paper appears in Wallach’s forthcoming book *Geometric invariant theory over the real and complex numbers*. New to this picture is the analog of the Kostant–Rallis theory of harmonic polynomials.

In this talk we will provide an overview of a family of identities from the theory of symmetric functions that reflect the structure of the harmonics for the Vinberg pair $(GL_{p+q+r}, GL_p \times GL_q \times GL_r)$ as $p, q, r \to \infty$. (Received September 20, 2015)